Write your name and student number in the top left corner of each page

1. Determine the z-transform of the following sequences. Wherever convenient, use the properties of the z-transform to make the solution easier

   (a) $x[n] = \left(\frac{1}{2}\right)^n \mu[-n]$

   (b) $x[n] = \left(\frac{1}{3}\right)^n \mu[n] + 4^n \mu[-n-1]$

   (c) $x[n] = n\left(\frac{1}{2}\right)^n \mu[n+1]$

2. Consider the discrete-time LTI system described by the following simple difference equation:

   $y[n] = x[n] - x[n-1]$

   (a) determine the impulse respone of this system, $h[n]$, and plot $h[n]$.
   (b) determine and write a closed-form expression for the DTFT, $H(e^{i\omega})$, of $h[n]$
   (c) plot the magnitude $|H(e^{i\omega})|$ over the range $-\pi < \omega < \pi$
   (d) plot the phase of $H(e^{i\omega})$ over $-\pi < \omega < \pi$

3. Given the second order band stop filter with transfer function

   $$H_{BS}(z) = \frac{\kappa(1 - 2\beta z^{-1} + z^{-2})}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$

   where α, β and γ are real constants with |α| < 1 and |β| < 1

   (a) determine α and β so that the filter has a notch at $\omega_0 = 0.3\pi$ and a band width of 0.3π
   (b) what is the quality factor of the filter
   (c) draw the magnitude response of $H_{BS}(z)$

4. Consider the discrete-time LTI system defined by the transfer function

   $$H(z) = \frac{20 - 24z^{-1} + 20z^{-2}}{(2 - z^{-1})(2 + 2z^{-1} + z^{-2})}$$

   (a) draw the pole-zero diagram of $H(z)$
   (b) given that the impulse response $h[n]$ of this system is causal, what is the ROC
   (c) draw a block diagram which implements this transfer function in cascade form using 1st and 2nd order sections.

5. The sequence of Fibonacci number $f[n]$ is a causale sequence defined by

$$f[n] = f[n-1] + f[n-2] \ , \ n \geq 2 \ \text{ with } f[0] = 0 \text{ and } f[1] = 1$$

(a) develop an exact formula to calculate $f[n]$ directly for any n

(b) show that $f[n]$ is the impulse response of a causal LTI system described by the difference equation $y[n] = y[n-1] + y[n-2] + x[n-1]$